

A FIELD METHOD FOR THE DETERMINATION OF INDEXES USED
IN SOIL WATER MOVEMENT EQUATIONS

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Abstract. The authors analysed the applicability of the Richards equation for the description of water movement in soil. The Richards equation was solved with the method of finite differences, using an overt schematic. The experiment was conducted on a grassland, in the surface layer of which TDR probes were placed for soil moisture measurement. It was demonstrated that using so-called calibration of the mono-dimensional model based on the Richards equation it is possible to determine van Genuchten indices characterizing the soil space under study. The results obtained from computer simulation were compared with the results of the field experiment.

Keywords: van Genuchten parameters, TDR technique, soil moisture

INTRODUCTION

Mathematical modelling of natural phenomena most frequently involves the use of non-linear differential equations. Necessary for their solution is parametrization of the space modelled, as accurate as possible. For the description of water movement in soil the Richards equation is used, and the modelled space in this case is a porous medium with strongly varied structure (Brandyk *et al.* 1993, Janik 2009, Sławiński *et al.* 2002, Sławiński 2003). Hence, giving the input conditions for the model requires numerous labour-intensive determinations. The objective of the work is to present a method for the determination of soil parameters (van Genuchten indexes) on the basis of direct measurements in the field.

MATERIAL AND METHOD

The field experiment was conducted on a grassland in the locality of Pszczew in the Wielkopolska Region (Poland). The experiment consisted in point-wise

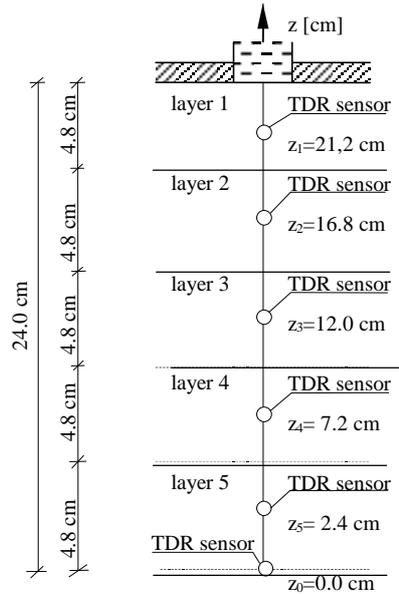


Fig. 1. Modelled soil space

irrigation of a section of soil and observation of changes in volumetric soil moisture beneath the irrigated point. The irrigation consisted in maintaining a constant column of water, $h_w = 10$ cm, in a cylinder of 10 cm in diameter (Fig. 1). Changes in the soil moisture were observed by means of a meter made at the Institute of Agrophysics, PAS, in Lublin (Malicki *et al.* 1992, Skierucha *et al.* 2004, Skierucha 2005). The distribution of the probes is also shown in Figure 1. In the study it was assumed that water movement caused by the irrigation was only in the vertical direction and only to the depth of 24 cm.

The water movement in the soil area under analysis was described by means of the Richards equation which, for a mono-dimensional space, assumes the form (Reinhard 2004):

$$C(h) \frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial z} \left[K(h) \frac{\partial \Phi}{\partial z} \right], \quad (1)$$

where: $C(h)$ – differentiable water capacity, $C = d\theta/dh$, h – matrix potential, cm H₂O, θ – moisture, m³ m⁻³, Φ – total potential, cm H₂O, $\Phi = h + z$, z – height, z – vertical coordinate, cm, $K(h)$ – hydraulic conductivity, cm min⁻¹.

For the determination of moisture distribution in the sample studied, the method of finite differences was used, digitising the modelled space (Fig. 1). The differential form of equation 1 for a mono-dimensional space (overt schematic) can be written as follows:

$$\frac{C_i^K}{\Delta t} (\Phi_i^{k+1} - \Phi_i^k) = \frac{1}{\Delta z} \left[K_{i+1/2}^k \frac{\Phi_{i+1}^k - \Phi_i^k}{\Delta z} - K_{i-1/2}^k \frac{\Phi_i^k - \Phi_{i-1}^k}{\Delta z} \right], \quad (2)$$

where: Δt – time step, min, Δz – spatial step, cm, k – time index, i – spatial index.

Moreover, for further calculations the following were adopted:

$$K_{i\pm 1/2} = \sqrt{K_i^k \cdot K_{i\pm 1}^k}. \quad (3)$$

For the determination of a relation corresponding to the curve of hydraulic conductivity the following formula was applied (Genuchten van 1980):

$$K(h) = \frac{K_s \left\{ 1 - (\alpha|h|)^{mn} [1 + (\alpha|h|^n)^{-m}]^2 \right\}}{[1 + (\alpha|h|^n)^{m/2}], \quad (4)$$

while the relation corresponding to the pF curve was determined from the formula:

$$\theta(h) = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha|h|^n)]^m}, \quad (5)$$

where: m, n, α – indices related to the type of soil, $m = 1 - 1/n$, θ_r – content of residual water, $\text{m}^3 \text{m}^{-3}$, θ_s – moisture in the full saturation zone, $\text{m}^3 \text{m}^{-3}$, K_s – infiltration index, $\text{cm} \cdot \text{min}^{-1}$.

For the solution of the equation it is necessary to determine the initial and boundary conditions. As the initial condition, the distribution of moisture in the modelled space at the start point of the experiment was adopted. The lower boundary condition was the time-variable distribution of moisture in layer 5, and the upper boundary condition – moisture distribution in layer 1. Therefore, the modelled area covered layers 2, 3 and 4. (Fig. 1). The simulation was conducted with a time step of $\Delta T = 5$ minutes, for a period of 240 minutes. The stability and convergence of the numeric solution was tested and verified through a numeric experiment.

RESULTS

Figure 2 presents changes in soil moisture in layers 1, 2, 4 and 5 in the course of the experiment. The Figure indicates correct response of moisture changes. In the initial period, the moisture increments in the higher layers were the greatest and occurred the fastest. Fig. 3 presents a comparison of the values of moisture obtained on the basis of computer simulation with values obtained through measurements for the layer of $z_2 = 16.8$ cm. The indices (θ , θ_s , K_s , n , α) affecting the shape and position of the line, related to the physical properties of the modelled space, were selected so that the moisture values – calculated and measured with the TDR meter – differ as little as possible. The procedure consists in error minimisation, i.e. selection of parameters of van Genuchten equations for the water retention curve and the hydraulic conductivity coefficient. This is so-called calibration of mono-dimensional model of water migration based on the Richards equation. As the criterion of fitting the sum of average value deviations (B_c) was adopted, calculated for each layer from the formula:

$$B_c = \sum_{i=2}^4 B_i, \quad (6)$$

where: B_c – sum of deviations of average values in layers 2 and 4, B_i – average deviation in i -th layer.

The deviation in i -th layer (B_i) was calculated from the formula:

$$B_i = \frac{1}{N} \sum_{k=1}^N |\theta_i^{k\,obl} - \theta_i^{k\,pom}|, \quad (7)$$

where: B_i – average deviation for i -th layer, i – layer number (spatial index), N – number of compared pairs of moisture values in the course of the experiment, $\theta_i^{k\,obl}$ – calculated moisture in i -th layer, at k -th time moment, $\text{m}^3 \text{m}^{-3}$, $\theta_i^{k\,pom}$ – measured moisture in i -th layer, at k -th time moment, $\text{m}^3 \text{m}^{-3}$.

As a result of identification procedure performed in the above manner, the summary average deviation B_C was reduced to $B_{Cmin} = 0.058$. The value of B_{Cmin} was obtained for: $\theta_r = 0.01 \text{ m}^3 \text{m}^{-3}$, $\theta_s = 0.365 \text{ m}^3 \text{m}^{-3}$, $K_s = 0.16 \text{ cm} \text{min}^{-1}$, $n = 1.659$, $\alpha = 0.278 \text{ m}^{-1}$.

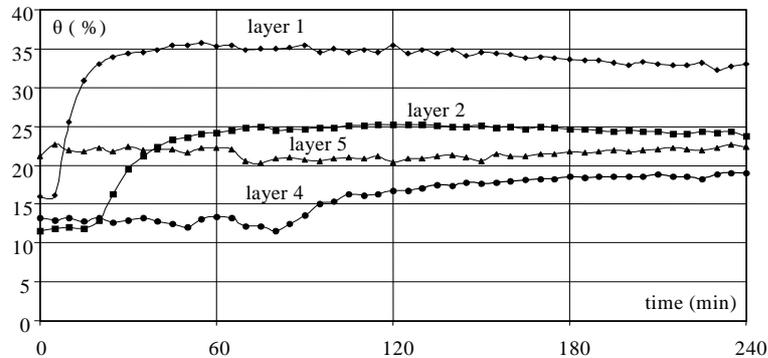


Fig. 2. Dynamics of moisture in measurement points 1, 2, 4, 5

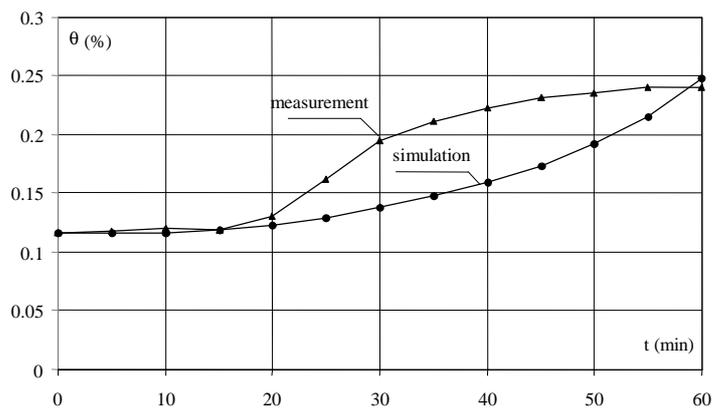


Fig. 3. Comparison of measured and calculated values of moisture in layer $z_2 = 16.8$ cm

The differences between the calculated and the measured values (Fig.3) may result from the lack of calibration of the TDR meter that should be performed individually for every medium in which measurements are taken (verbal information, Skierucha W.). Moreover, the Richards equation (written in the form 1) is true only when the medium is homogeneous, i.e. the hydraulic conductivity of the material for water $K(h)$ and the hydraulic potential Φ , related to the water content, are constant for each point in the medium studied. Also, the medium should be isotropic, i.e. its hydraulic conductivity should not depend on the direction of water movement. In the equation it is also assumed that θ_s is approximately equal to porosity, and that the volume of solid particles is invariable in time. These assumptions were not verified in the study reported herein.

CONCLUSION

The experiment performed and computer simulations permitted the demonstration that by using so-called calibration of mono-dimensional model based on the Richards equation it is possible to determine van Genuchten indices characterizing the soil studied.

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POŁOWA METODA WYZNACZANIA WSPÓŁCZYNNIKÓW
WYKORZYSTYWANYCH W RÓWNANIACH RUCHU WODY GLEBOWEJ

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Streszczenie. W pracy przeanalizowano przydatność zastosowania równania Richardsa do opisu ruchu wody w glebie. Równanie Richardsa rozwiązano metodą różnic skończonych, stosując schemat jawny. Eksperyment przeprowadzono na użytku łąkowym, w którego wierzchniej warstwie umieszczono czujniki TDR do pomiaru wilgotności. Wykazano, że stosując tzw. kalibrację jednowymiarowego modelu opartego o równanie Richardsa można wyznaczyć współczynniki van Genuchtena charakteryzujące badaną przestrzeń. Wyniki uzyskane z symulacji komputerowej porównano z wynikami eksperymentu.

Słowa kluczowe: parametry van Genuchtena, technika TDR, wilgotność gleby